RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. FOURTH SEMESTER EXAMINATION, JUNE 2022

SECOND YEAR (BATCH 2020-23)

Date : 23/06/2022 **MATHEMATICS (Honours)**

Time : 11.00 am – 1.00 pm **Paper : IX [CC 9]** Full Marks : 50

Group – A

Answer **any three** questions:

1. Find a complete integral of

   $y(1+x)p = x(1+y)q$ [5]

2. Find the integral surface of the pde

   $(x-y)p+(y-x-z)q = z$

   through the hyperbolid $xy = x+y\,;z=1$ [5]

3. Find a complete integral and singular solution (if any) for

   $pz = \left(p^2+q^2\right)x$ [5]

4. Solve $z\left(x^2+y^2\right) = x^2(y-x)q + y^2(x-y)p$ [5]

5. Solve the following pde problem using Charpit's method.

   $u_{x_1} + u_{x_2} = x_2 u$ on $\Omega = \mathbb{R}^2$

   with boundary condition

   $u(x_1, x_2) = x_1^2$ on $\Gamma = \mathbb{R}\times\{0\}$ [5]

Group – B

Answer **all** questions, maximum you get 10 marks.

6. Find $L\left\{e^t \sin^2 t\right\}$. [2]

7. Prove that $L\left\{\dfrac{\sin t}{t}\right\} = \tan^{-1}\dfrac{1}{p}$. Hence find $L\left\{\dfrac{\sin at}{t}\right\}$. Does the Laplace transform of $\dfrac{\cos at}{t}$ exist? Justify your answer. [1+1+0.5+1.5]

8. Find $L^{-1}\left\{\dfrac{3p+7}{p^2-2p-3}\right\}$. [2]

9. Solve $\dfrac{d^2x}{dt^2} + m^2x = a\cos nt,\ \ t>0;\ x = x_0$ and $\dfrac{dx}{dt} = x_1$ when $t=0,\ n\neq m$ by using Laplace transform technique. [4]

Group – C

Answer **any five** questions: [5×5]

10. Prove that the product of k consecutive integers is divisible by $k!$, where $k \in \mathbb{N}$. [5]

11. Let $a,b,c \in \mathbb{Z}$. The prove the following.

    a) If $a|c$ and $b|c$ with $\gcd(a,b)=1$ then $ab|c$

    b) If $a|bc$ with $\gcd(a,b)=1$ then $a|c$. [3+2]

12. Determine all solutions in the positive integers of the Diophantine equation $18x+5y=48$. [5]

13. Prove that there are infinitely many primes of the form 4n+3. [5]

14. Solve the following sets of simultaneous congruences

    $x \equiv 1 \pmod 3$, $x \equiv 2 \pmod 5$, and $x \equiv 3 \pmod 7$ [5]

15. Let $N = a_n 10^n + a_{n-1}10^{n-1} + \ldots\ldots\ldots + a_1 10 + a_0$ be the decimal expansion of positive integer N, $0 \le a_k < 10$ and let $T = a_0 - a_1 + a_2 - a_3 + \ldots\ldots\ldots + (-1)^n a_n$. Prove that $11|N$ if and only if $11|T$. [5]

16. Prove that the sum $\frac{1}{p_1} + \frac{1}{p_2} + \ldots\ldots\ldots + \frac{1}{p_n}$ is never an integer where p_n is the n^{th} prime. [5]

17. Compute the following.

    a) $\phi(480)$

    b) $\sigma(96)$ and $\tau(100)$

    Where ϕ is the Euler's phi function, also τ and σ are divisor function and sum of divisors function respectively. [2+3]

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